Chapter Twelve

Simple Simultaneous Equations with Two Variables

For solving the mathematical problems, the most important topic of Algebra is equation. In classes VI and VII, we have got the idea of simple equation and have known how to solve the simple equation with one variable. In class VIII, we have solved the simple simultaneous equations by the methods of substitution and elimination and by graphs. We have also learnt how to form and solve simple simultaneous equations related to real-life, problems. In this chapter, the idea of simple simultaneous equations have been expanded and new methods of solution have been discussed. Besides, in this chapter, solution by graphs and formation of simultaneous equations related to real life problems and their solutions have been discussed in detail.

At the end of the chapter, the students will be able to –

- Verify the consistency of simple simultaneous equations with two variables.
- Verify the mutual dependence of two simple simultaneous equations with two variables
- Explain the method of cross-multiplication
- Form and solve simultaneous equations related to real life mathematical problems
- Solve the simultaneous equations with two variables by graphs.

12·1 Simple simultaneous equations.

Simple simultaneous equations means two simple equations with two variables when they are presented together and the two variables are of same characteristics. Such two equations together are also called system of simple equations. In class VIII, we have solved such system of equations and learnt to form and solve simultaneous equations related to real life problems. In this chapter, these have been discussed in more details.

First, we consider the equation $2x + y = 12$. This is a simple equation with two variables.

In the equation, can we get such values of x and y on the left hand side for which the sum of twice the first with the second will be equal to 12 of the right hand side ; that is, the equation will be satisfied by those two values ?

Now, we fill in the following chart from the equation $2x + y = 12$:

Value of x	Value of y	Value of L.H.S $(2x+y)$	R.H.S
-2	16	$-4+16=12$	12
0	12	$0+12=12$	12
3	6	$6+6=12$	12
5	2	$10+2=12$	12
....	.....	..... $=12$	12

The equation has infinite number of solutions. Among those, four solutions are $(-2,16),\ (0,12),\ (3,6)$ and $(5,2)$.

Again, we fill in the following chart from another equation $x-y=3$:

Value of x	Values of y	Value of L.H.S $(x-y)$	R.H.S
-2	-5	$-2+5=3$	3
0	-3	$0+3=3$	3
3	0	$3-0=3$	3
5	2	$5-2=3$	3
....	.....	..... $=3$	3

The equation has infinite number of solutions. Among those, four solutions are $(-2,-5),\ (0,-3),\ (3,0)$ and $(5,2)$.

If the two equations discussed above are considered together a system, both the equations will be satisfied simultaneously only by (5, 2). Both the equations will not be satisfied simultaneously by any other values.

Therefore, the solution of the system of equations $2x+y=12$ and $x-y=3$ is $(x,y)=(5,2)$

> **Activity :** Write down five solutions for each of the two equations $x-2y+1=0$ and $2x+y-3=0$ so that among the solutions, the common solutions also exists.

12·2 Conformability for the solution of simple simultaneous equations with two variables.

(a) As discussed earlier, the system of equations $\left.\begin{array}{r}2x+y=12\\ x-y=3\end{array}\right\}$ have unique (only one) solution. Such system of equations are called consistent. Comparing the coefficient of x and y (taking the ratio of the coefficients) of the two equations, we get, $\frac{2}{1}\neq\frac{1}{-1}$; any equation of the system of equations cannot be expressed in terms of the other. That is why, such system of equations are called mutually independent. In the case of consistent and mutually independent system of equations, the ratios are not equal. In this case, the constant terms need not to be compared.

(b) Now we shall consider the system of equations $\left.\begin{aligned} 2x - y &= 6 \\ 4x - 2y &= 12 \end{aligned}\right\}$. Will this two equations be solved ?

Here, if both sides of first equation are multiplied by 2, we shall get the second equation. Again, if both sides of second equation are divided by 2, we shall get the first equation. That is, the two equations are mutually dependent.

We know, first equation has infinite number of solutions. So, 2nd equation has also the same infinite number of solutions. Such system of equations are called consistent and mutually dependent. Such system of equations have infinite number of solutions.

Here, comparing the coefficients of x, y and the constant terms of the two equations, we get, $\frac{2}{4} = \frac{-1}{-2} = \frac{6}{12}\left(= \frac{1}{2}\right)$.

That is, in the case of the system of such simultaneous equations, the ratios become equal.

(c) Now, we shall try to solve the system of equations $\left.\begin{aligned} 2x + y &= 12 \\ 4x + 2y &= 5 \end{aligned}\right\}$.

Here, multiplying both sides of first equation by 2, we get, $4x + 2y = 24$

second equation is $4x + 2y = 5$

subtracting, $0 = 19$, which is impossible.

So, we can say, such system of equations cannot be solved. Such system of equations are inconsistent and mutually independent. Such system of equations have no solution.

Here, comparing the coefficients of x, y and constant terms from the two equations, we get, $\frac{2}{4} = \frac{1}{2} \neq \frac{12}{5}$. That is, in case of the system of inconsistent and mutually independent equations ratios of the coefficients of the variables are not equal to the ratio of the constant terms. Generally, conditions for comformability of two simple simultaneous equations, such as, $\left.\begin{aligned} a_1x + b_1y &= c_1 \\ a_2x + b_2y &= c_2 \end{aligned}\right\}$ are given in the chart below :

	system of equations	**Comparison of coeff.and const. terms**	**consistent/ incon sistent**	**mutually dependent/ independent**	**has solution (how many) / no.**
(i)	$a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	consistent	independent	Yes (only one)
(ii)	$a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	consistent	dependent	Yes (infinite numbers)
(iii)	$a_1x + b_1y = c_1$ $a_2x + b_2y = c_2$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	inconsistent	independent	No

Now, if there is no constant terms in both the equations of a system of equations ; i.e., $c_1 = c_2 = 0$, if with reference to the above discussion from (i), if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ the system of equations are always consistent and independent of each other. In that case, there will be only one (unique) solution.

From (*ii*) and (*iii*) if $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the system of equations are consistent and dependent of each other. In that case, there will be infinite number of solutions.

Example : Explain whether the following sy stem of equations are consistent / inconsistent, dependent/ independent of each other and indicate the number of solutions in each case.

(a) $x + 3y = 1$, $2x + 6y = 2$ (b) $2x - 5y = 3$, $x + 3y = 1$ (c) $3x - 5y = 7$, $6x - 10y = 15$

Solution :

(a) Given system of equations are : $\left.\begin{aligned} x + 3y &= 1 \\ 2x + 6y &= 2 \end{aligned}\right\}$

Ratio of the coefficients of x is $\frac{1}{2}$

Ratio of the coefficients of y is $\frac{3}{6}$ or $\frac{1}{2}$

Ratio of constant terms is $\frac{1}{2}$

$\therefore \quad \frac{1}{2} = \frac{3}{6} = \frac{1}{2}$

Therefore, the system of equations are consistent and mutually dependent. The system of equations have infinite number of solutions.

(b) Given system of equations are : $\left.\begin{aligned} 2x - 5y &= 3 \\ x + 3y &= 1 \end{aligned}\right\}$

Ratio of the coefficients of x is $\frac{2}{1}$

Ratio of the coefficients of y is $\frac{-5}{3}$

$\therefore$ we have, $\frac{2}{1} \neq \frac{-5}{3}$

Therefore, the system of equations are consistent and mutually independent. The system of equations have only one (unique) solution.

(c) Given system of equations are : $3x - 5y = 7$

$6x - 10y = 15$

Ratio of the coefficients of x is $\frac{3}{6}$ or $\frac{1}{2}$

Ratio of the coefficients of y is $\frac{-5}{-10}$ or $\frac{1}{2}$

ratio of the constant terms is $\frac{7}{15}$

$\therefore$ we get, $\frac{3}{6} = \frac{-5}{-10} \neq \frac{7}{15}$

Therefore, the system of equations are inconsistent and mutually independent. The system of equations have no solution.

Activity : Verify whether the system of equations $x - 2y + 1 = 0, 2x + y - 3 = 0$ are consistent and dependent and indicate how many solutions the system of equations may have.

Exercise 12·1

Mention with arguments, whether the following simple simultaneous equations are consistent/inconsistent, mutually dependen t/independent and indicate the number of solutions :

1. $x - y = 4$
   $x + y = 10$
2. $2x + y = 3$
   $4x + 2y = 6$
3. $x - y - 4 = 0$
   $3x - 3y - 10 = 0$
4. $3x + 2y = 0$
   $6x + 4y = 0$
5. $3x + 2y = 0$
   $9x - 6y = 0$
6. $5x - 2y - 16 = 0$
   $3x - \frac{6}{5}y = 2$
7. $-\frac{1}{2}x + y = -1$
   $x - 2y = 2$
8. $-\frac{1}{2}x - y = 0$
   $x - 2y = 0$
9. $-\frac{1}{2}x + y = -1$
   $x + y = 5$
10. $ax - cy = 0$
    $cx - ay = c^2 - a^2$.

12·3 Solution of simple sumultaneous equations

We shall discuss the solutions of only the consistent and independent simple simultaneous equations. Such system of equation has only one (unique) solution.

Here, four methods of solutions are discussed :

(1) Method of substitution, (2) Method of elimination (3) Method of cross-multiplication (4) Graphical method.

In class VII, we have known how to solve by the methods of substitution and elimination. Here, examples of one for each of these two methods are given.

Example 1. Solve by the method of substitution :

$$2x + y = 8$$
$$3x - 2y = 5$$

Solution : Given equations are :

$$2x + y = 8 \text{.............} (1)$$
$$3x - 2y = 5 \text{...........} (2)$$

From equation (1), $y = 8 - 2x$.........(3)

Putting the value of y form equation (3) in equation (2), we get

$3x - 2(8 - 2x) = 5$

or, $3x - 16 + 4x = 5$

or, $3x + 4x = 5 + 16$

or, $7x = 21$

or, $x = 3$

Putting the value of x in equation (3)

$y = 8 - 2 \times 3$

$= 8 - 6$

$= 2$

$\therefore$ Solution $(x, y) = (3, 2)$

Solution by the method of substitution :

Conveniently, from any of the two equations, value of one variable is expressed in terms of the other variable and putting the obtained value in the other equation, we shall get an equation with one variable. Solving this equation, value of the variable can be found. This value can be put in any of the equations. But, if it is put in the equation in which one variable has been expressed in terms of the other variable, the solution will be easier. From this equation, value of the other variable will be found.

Example 2. Solve by the method of elimination : $2x + y = 8$

$3x - 2y = 5$

[N.B. : To show the difference between the methods of substitution and elimination, same equations of example 1 have been taken in this example 2]

Solution : Given equations are $2x + y = 8$.............(1)

$3x - 2y = 5$............(2)

Multiplying both sides of equation (1) by 2, $4x + 2y = 16$.............(3)

equation (2) is $3x - 2y = 5$...........(2)

Adding (3) and (2), $7x = 21$ or $x = 3$.

Putting the value of x in equation (1), we get

$2 \times 3 + y = 8$

or, $y = 8 - 6$

or, $y = 2$

$\therefore$ Solution $(x, y) = (3, 2)$

Solution by the method of elimination :

Conveniently, one equation or both equations are multiplied by such a number so that after multiplication, absolute value of the coefficients of the same variable become equal. Then as per need, if the equations are added or subtracted, the variable with equal coefficient will be eliminated. Then, solving the obtained equation, the value of the existing variable will be found. If that value is put conveniently in any of the given equations, value of the other variable will be found.

(3) Method of cross-multiplication :

We consider the following two equations :

$$a_1x + b_1y + c_1 = 0 \text{..........}(1)$$

$$a_2x + b_2y + c_2 = 0 \text{..........}(2)$$

Multiplying equation (1) by b_2 and equation (2) by b_1, we get,

$$a_1b_2x + b_1b_2y + b_2c_1 = 0 \text{.........}(3)$$

$$a_2b_1x + b_1b_2y + b_1c_2 = 0 \text{.........}(4)$$

Subtracting equation (4) from equation (3), we get

$$(a_1b_2 - a_2b_1)x + b_2c_1 - b_1c_2 = 0$$

or, $(a_1b_2 - a_2b_1)x = b_1c_2 - b_2c_1$

or, $\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{1}{a_1b_2 - a_2b_1}$ ....(5)

Again, multiplying equation (1) by a_2 and equation (2) by a_1, we get,

$$a_1a_2x + a_2b_1y + c_1a_2 = 0 \text{.........}(6)$$

$$a_1a_2x + a_1b_2y + c_2a_1 = 0 \text{.........}(7)$$

Subtracting equation (7) from equation (6), we get

$$(a_2b_1 - a_1b_2)y + c_1a_2 - c_2a_1 = 0$$

or, $-(a_1b_2 - a_2b_1)y = -(c_1a_2 - c_2a_1)$

or, $\dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$ ....(8)

From (5) and (8) we get,

$$\boxed{\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}}$$

From such relation between x and y, the technique of finding their values is called the method of cross-multiplication.

From the above relation between x and y, we get,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{1}{a_1b_2 - a_2b_1}, \quad \text{or} \quad x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}$$

Again, $\dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$, or $y = \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}$

$\therefore$ The solution of the given equations : $(x, y) = \left(\dfrac{b_1c_2 - b_2c_1}{a_1b_2 - a_2b_1}, \dfrac{c_1a_2 - c_2a_1}{a_1b_2 - a_2b_1}\right)$

We observe :

Equations	Relation between x and y	Illustration
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\dfrac{x}{b_1c_2 - b_2c_1} = \dfrac{y}{c_1a_2 - c_2a_1} = \dfrac{1}{a_1b_2 - a_2b_1}$	$x \quad y \quad 1$ $a_1 \mid b_1 \quad c_1 \quad a_1 \quad b_1$ $a_2 \mid b_2 \quad c_2 \quad a_2 \quad b_2$

[N.B. : The method of cross-multiplication can also be applied by keeping the constant terms of both equations on the right hand side. In that case, changes of sign will occur ; but the solution will remain same.]

Activity : If the system of equations

$\left.\begin{array}{l} 4x - y - 7 = 0 \\ 3x + y \quad\; = 0 \end{array}\right\}$ are expressed as the system of equations $\left.\begin{array}{l} a_1x + b_1y + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array}\right\}$,

find the values of $a_1, b_1, c_1, a_2, b_2, c_2$.

Example 3. Solve by the method of cross-multiplication : $6x - y = 1$
$3x + 2y = 13$

Solution : Making the right hand side of the equations 0 (zero) by transposition, we get,

$6x - y - 1 = 0$

$3x + 2y - 13 = 0$

comparing the equations with

$\left.\begin{array}{l} a_1x + b_1y_2 + c_1 = 0 \\ a_2x + b_2y + c_2 = 0 \end{array}\right\}$ respectively,

we get, $a_1 = 6, b_1 = -1, c_1 = -1$

$a_2 = 3, b_2 = 2, c_2 = -13$

By the method of cross-multiplication, we get,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$

Illustration :

$x \quad y \quad 1$

$a_1 \mid b_1 \quad c_1 \quad a_1 \quad b_1$

$a_2 \mid b_2 \quad c_2 \quad a_2 \quad b_2$

or $\frac{x}{(-1)\times(-13)-2\times(-1)}=\frac{y}{(-1)\times 3-(-13)\times 6}=\frac{1}{6\times 2-3\times(-1)}$

or $\frac{x}{13+2}=\frac{y}{-3+78}=\frac{1}{12+3}$

or $\frac{x}{15}=\frac{y}{75}=\frac{1}{15}$

$$\begin{array}{c|cccc} & \underline{x} & \underline{y} & \underline{1} & \\ 6 & -1 & -1 & 6 & -1 \\ 3 & 2 & -13 & 3 & 2 \end{array}$$

$\therefore\ \frac{x}{15}=\frac{1}{15}$, or $x=\frac{15}{15}=1$

Again, $\frac{y}{75}=\frac{1}{15}$, or $y=\frac{75}{15}=5$

$\therefore$ Solution $(x, y)=(1,5)$

Example 4. Solve by the method of cross-multiplication : $3x-4y=0$
$2x-3y=-1$

Solution : Given equations are :

$\left.\begin{array}{l}3x-4y=0\\2x-3y=-1\end{array}\right\}$ or $\left.\begin{array}{l}3x-4y+0=0\\2x-3y+1=0\end{array}\right\}$

By the method of cross-multiplication, we get,

$\frac{x}{-4\times 1-(-3)\times 0}=\frac{y}{0\times 2-1\times 3}=\frac{1}{3\times(-3)-2\times(-4)}$

$$\begin{array}{c|cccc} & \underline{x} & \underline{y} & \underline{1} & \\ 3 & -4 & 0 & 3 & -4 \\ 2 & -3 & 1 & 2 & -3 \end{array}$$

or $\frac{x}{-4+0}=\frac{y}{0-3}=\frac{1}{-9+8}$

or $\frac{x}{-4}=\frac{y}{-3}=\frac{1}{-1}$

or $\frac{x}{4}=\frac{y}{3}=\frac{1}{1}$

$\therefore\ \frac{x}{4}=\frac{1}{1}$, or $x=4$

Again, $\frac{y}{3}=\frac{1}{1}$, or $y=3$

$\therefore$ Solution $(x, y)=(4,3)$.

Example 5. Solve by the method of cross-multiplication : $\frac{x}{2}+\frac{y}{3}=8$
$\frac{5x}{4}-3y=-3$

Solution : Arranging the given equations in the form $ax+by+c=0$, we get,

$\frac{x}{2}+\frac{y}{3}=8$

or $\frac{3x+2y}{6}=8$

or $3x+2y-48=0$

Again, $\frac{5x}{4}-3y=-3$

or $\frac{5x-12y}{4}=-3$

or $5x-12y+12=0$

$\therefore$ the given equations are : $3x+2y-48=0$

$5x-12y+12=0$

By the method of cross-multiplication, we get,

$$\frac{x}{2\times12-(-12)\times(-48)}=\frac{y}{(-48)\times5-12\times3}=\frac{1}{3\times(-12)-5\times2}$$

	x	y	1	
3	2	−48	3	2
5	−12	12	5	−12

or $\frac{x}{24-576}=\frac{y}{-240-36}=\frac{1}{-36-10}$

or $\frac{x}{-552}=\frac{y}{-276}=\frac{1}{-46}$

or $\frac{x}{552}=\frac{y}{276}=\frac{1}{46}$

$\therefore \frac{x}{552}=\frac{1}{46}$ or, $x=\frac{552}{46}=12$

Again, $\frac{y}{276}=\frac{1}{46}$, or $y=\frac{276}{46}=6$

$\therefore$ Solution $(x,y)=(12,6)$

Verification of the correctness of the solution :

Putting the values of x and y in given equations, we get,

In 1st equation, L.H.S $=\frac{x}{2}+\frac{y}{3}=\frac{12}{2}+\frac{6}{3}=6+2$

$=8=$ R.H.S

In 2nd equation, L.H.S $=\frac{5x}{4}-3y=\frac{5\times12}{4}-3\times6$

$=15-18=-3=$ R.H.S

$\therefore$ the solution is correct.

Example 6. Solve by the method of cross-multiplication : $ax-by=ab=bx-ay$.

Solution : Given equations are

$$\left.\begin{aligned}ax-by&=ab\\bx-ay&=ab\end{aligned}\right\}\text{ or, }\left.\begin{aligned}ax-by-ab&=0\\bx-ay-ab&=0\end{aligned}\right\}$$

By the method of cross-multiplication, we get,

$$\therefore \frac{x}{(-b)\times(-ab)-(-a)(-ab)} = \frac{y}{(-ab)\times b-(-ab)\times a} = \frac{1}{a\times(-a)-b\times(-b)}$$

	x	y	1	
a	$-b$	$-ab$	a	$-b$
b	$-a$	$-ab$	b	$-a$

or $\dfrac{x}{ab^2-a^2b} = \dfrac{y}{-ab^2+a^2b} = \dfrac{1}{-a^2+b^2}$

or $\dfrac{x}{-ab(a-b)} = \dfrac{y}{ab(a-b)} = \dfrac{1}{-(a+b)(a-b)}$

or $\dfrac{x}{ab(a-b)} = \dfrac{y}{-ab(a-b)} = \dfrac{1}{(a+b)(a-b)}$

$\therefore \dfrac{x}{ab(a-b)} = \dfrac{1}{(a+b)(a-b)}$, or $x = \dfrac{ab(a-b)}{(a+b)(a-b)} = \dfrac{ab}{a+b}$

Again, $\dfrac{y}{-ab(a-b)} = \dfrac{1}{(a+b)(a-b)}$, or $y = \dfrac{-ab(a-b)}{(a+b)(a-b)} = \dfrac{-ab}{a+b}$

$\therefore (x, y) = \left(\dfrac{ab}{a+b}, \dfrac{-ab}{a+b}\right)$

Exercise 12·2

Solve by the method of substitution (1 -3) :

1. $7x-3y=31$, $9x-5y=41$
2. $\dfrac{x}{2}+\dfrac{y}{3}=1$, $\dfrac{x}{3}+\dfrac{y}{2}=1$
3. $\dfrac{x}{a}+\dfrac{y}{b}=2$, $ax+by=a^2+b^2$

Solve by the method of elimination (4 -6) :

4. $7x-3y=31$, $9x-5y=41$
5. $7x-8y=-9$, $5x-4y=-3$
6. $ax+by=c$, $a^2x+b^2y=c^2$

Solve by the method of cross-multiplication (7 -15) :

7. $2x+3y+5=0$, $4x+7y+6=0$
8. $3x-5y+9=0$, $5x-3y-1=0$
9. $x+2y=7$, $2x-3y=0$
10. $4x+3y=-12$, $2x=5$
11. $-7x+8y=9$, $5x-4y=-3$
12. $3x-y-7=0=2x+y-3$
13. $ax+by=a^2+b^2$, $2bx-ay=ab$
14. $y(3+x)=x(6+y)$, $3(3+x)=5(y-1)$
15. $(x+7)(y-3)+7=(y+3)(x-1)+5$

12·4 Solution by graphical method

In a simple equation with two variables, the relation of existing variables x and y can be expressed by picture. This picture is called the graphs of that relation. In the graph of such equation, there exist infinite number of points. Plotting a few such points, if they are joined with each other, we shall get the graph.

Each of a simple simultaneous equations has infinite number of solutions. Graph of each equation is a straight line. Coordinates of each point of the straght line satisfies the equation. To indicate a graph, two or more than two points are necessary.

Now we shall try to solve graphically the following system of equations :

$$2x + y = 3..........(1)$$
$$4x + 2y = 6........(2)$$

From equation (1), we get, $y = 3 - 2x$.

Taking some values of x in the equation, we find the corresponding values of y and make the adjoining table :

x	-1	0	3
y	5	3	-3

$\therefore$ three points on the graph of the equation are : $(-1, 5), (0, 3)$ and $(3, -3)$ |

Again, from equation (2), we get, $2y = 6 - 4x$ or, $y = \dfrac{6-4x}{2}$

Taking some values of x in the equations, we find the the corresponding values of y and make the adjoining table :

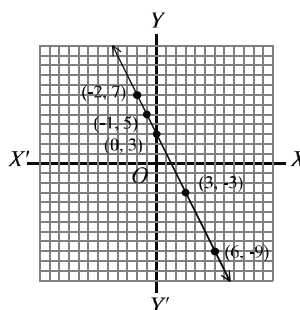
x	-2	0	6
y	7	3	-9

$\therefore$ three points on the graph of the equation are : $(-2, 7), (0, 3)$ and $(6, -9)$

In a graph paper let XOX' and YOY' be respectively the x axis and y axis and O is the origin.

We take each side of smallest squares of the graph paper as unit along with both axes. Now, we plot the points $(-1, 5), (0, 3)$ and $(3, -3)$ obtained from equation (1) and join them each other. The graph is a straight line.

Again, we plot the points $(-2, 7), (0, 3)$ and $(6, -9)$ obtained from equation (2) and join them each other. In this case also the graph is a straight line. But we observe that the two straight lines coincide and they have turned into the one straight line. Again, if both sides of equation (2) are divided by 2, we get he equation (1). That is why the graphs of the two equations coincide.

Here, the system of equations, $\left.\begin{array}{l} 2x + y = 3..........(1) \\ 4x + 2y = 6........(2) \end{array}\right\}$ are consistent and mutually dependent. Such system of equations have infinite number of solutions and its graph is a straight line.

Now, we shall try to solve the system of equations : $2x - y = 4$..........(1)

$4x - 2y = 12$......(2)

From equation (1), we get, $y = 2x - 4$.

Taking some values of x in the equation, we find the corresponding values of y and make the adjoining table :

x	-1	0	4
y	-6	-4	4

$\therefore$ three points on the graph of the equation are : $(-1, -6), (0, -4), (4, 4)$.

Again, from equation (2), we get,

$4x - 2y = 12$, or $2x - y = 6$

or $y = 2x - 6$

Taking some values of x in the equation, we find the the corresponding values of y and make the adjoining table :

x	0	3	6
y	-6	0	6

$\therefore$ three points on the graph of the equation are : $(0, -6), (3, 0), (6, 6)$ |

In graph paper let XOX' and YOY' be respectively x axis and y axis and O is the origin.

Taking each side of smallest squares in the graph paper as unit, we plot the points $(-1, -6), (0, -4)$ and $(4, 4)$ obtained from equation (1) and join them each other. The graph is a straight line.

Again, we plot the points $(0, -6), (3, 0), (6, 6)$ obtained from equation (2) and join them each other. In this case also the graph is a straight line.

We observe in the graph, though each of the given equations has separately infinite number of solutions, they have no common solution as system of simultaneous equations. Further, we observe that the graphs of the two equations are straight lines parallel to each other. That is, the lines will never intersect each other. Therefore, there will be no common point of the lines. In this case we say, such system of equations have no solution. We know, such system of equations are inconsistent and independent of each other.

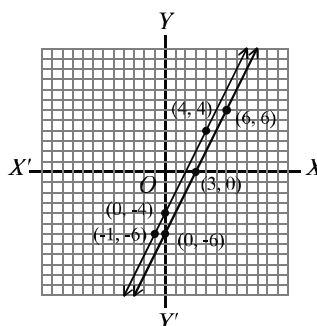
Now, we shall solve the system of two consistent and independent equations by graphs. Graphs of two such equations with two variables intersect at a point. Both the equations will be satisfied by the coordinates of that point. The very coordinates of the point of intersection will be the solution of the two equations.

Example 7. Solve and show the solution in graph : $2x + y = 8$

$3x - 2y = 5$

Solution : : Given two equations are : $2x + y - 8 = 0$...........(1)

$3x - 2y - 5 = 0$............(2)

By the method of cross-multiplication, we get,

$$\frac{x}{1\times(-5)-(-2)\times(-8)}=\frac{y}{(-8)\times3-(-5)\times2}=\frac{1}{2(-2)-3\times1}$$

or $\frac{x}{-5-16}=\frac{y}{-24+10}=\frac{1}{-4-3}$

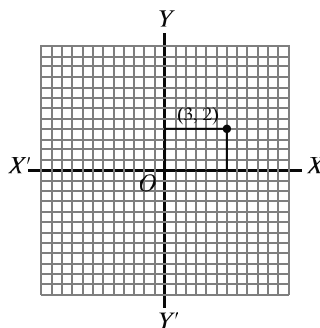
or $\frac{x}{-21}=\frac{y}{-14}=\frac{1}{-7}$

or $\frac{x}{21}=\frac{y}{14}=\frac{1}{7}$

$\therefore \frac{x}{21}=\frac{1}{7}$, or $x=\frac{21}{7}=3$

Again, $\frac{y}{14}=\frac{1}{7}$, or $y=\frac{14}{7}=2$

$\therefore$ solution is $(x,y)=(3,2)$

Let XOX' and YOY' be x axis and y axis respectively and O, the origin.
Taking each two sides of the smallest squares along with both axes of the graph paper as one unit, we plot the point (3, 2).

Example 8. Solve with the help of graphs :

$$3x-y=3$$
$$5x+y=21$$

Solution : : Given equations are : $3x-y=3$..........(1)
$5x+y=21$........(2)

From equation (1), we get, $3x-y=3$, or $y=3x-3$
Taking some values of x in equation (1), we get corresponding values of y and make the table beside :

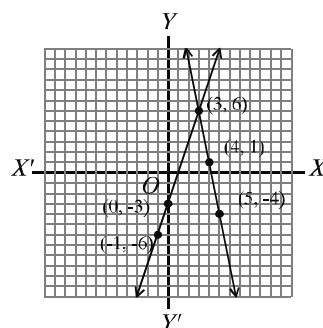
x	-1	0	3
y	-6	-3	6

$\therefore$ three points on the graph of the equation are : $(-1,-6),(0,-3),(3,6)$

Again, from equation (2), we get, $5x+y=21$, or $y=21-5x$
Taking some values of x in equation (2), we find the corresponding values of y and make the adjoining table :

x	3	4	5
y	6	1	-4

$\therefore$ three points on the graph of the equation are : $(3,6),(4,1),(5,-4)$.

Let XOX' and YOY' be respectively x axis and y axis and O be the origin.
We take each side of the smallest square of the graph paper as unit.

Now, we plot the points $(-1,-6),(0,-3),(3,6)$ obtained from equation (1) and join them successively. The graph is a straight line. Similarly, we plot the points $(3,6),(4,1),(5,-4)$ obtained from equation (2) and join them successively. In this case also the graph is a straight line. Let the two straight lines intersect each other at P. It is seen from the picture that the coordinates of P are $(3,6)$.

$\therefore$ solution is $(x, y)=(3,6)$

Example 9. Solve by graphical method : $2x+5y=-14$

$4x-5y=17$

Solution : Given equations are : $2x+5y=-14$...........(1)

$4x-5y=17$........(2)

From equation (1), we get, $5y=-14-2x$, or $y=\frac{-2x-14}{5}$

x	3	$\frac{1}{2}$	-2
y	-4	-3	-2

Taking some convenient values of x in the equation, we find the corresponding values of y and make the adjoining table :

$\therefore$ three points on the graph of the equation are :

$(3,-4),\left(\frac{1}{2},-3\right),(-2,-2)$.

Again, from equation (2), $5y=4x-17$, or $y=\frac{4x-17}{5}$

x	3	$\frac{1}{2}$	-2
y	-1	-3	-5

Taking some convenient values of x in the equation (2), we find the corresponding values of y and make the adjoining table :

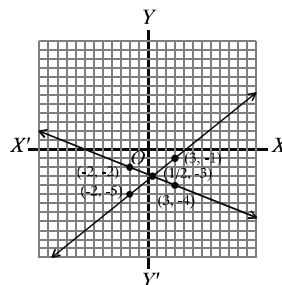
$\therefore$ three points on the graph of the equation are : $(3,1),\left(\frac{1}{2},-3\right),(-2,-5)$.

Let XOX' and YOY' be x axis and y axis respectively and O, the origin.

We take each two sides of the smallest squares as unit along with both axes.

Now, we plot the points $(3,-4),\left(\frac{1}{2},-3\right)$ and $(-2,-2)$ obtained from equation (1) in the graph paper and join them each other. The graph is a straight line. Similarly, we plot the points $(3,-1),\left(\frac{1}{2},-3\right),(-2,-5)$ obtained from equation (2) and join them each other. The graph is a straight line.

Let the straight lines intersect at P. It is seen from the graph, coordinates of P are $\left(\frac{1}{2},-3\right)$.

$\therefore$ solution is $(x, y)=\left(\frac{1}{2},-3\right)$

Example 10. Solve with the help of graphs : $3-\frac{3}{2}x=8-4x$

Solution : Given equation is $3-\frac{3}{2}x=8-4x$

Let, $y=3-\frac{3}{2}x=8-4x$

$\therefore \quad y=3-\frac{3}{2}x$.........(1)

And, $y=8-4x$.........(2)

Now, Taking some values of x in equation (1), we find the corresponding values of y and make the adjoining table :

x	-2	0	2
y	6	3	0

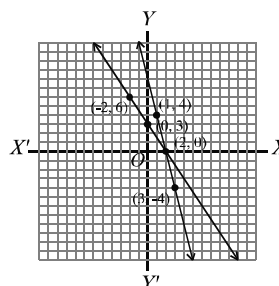
$\therefore$ three points on the graph of the equation (1) are : $(-2,6),(0,3),(2,0)$

Again, taking some values of x in equation (2), we find the corresponding values of y and make the adjoining table :

x	1	2	3
y	4	0	-4

$\therefore$ three points on the graph of the equation (2) are : $(1,4),(2,0),(3,-4)$

Let XOX' and YOY' be x axis, y axis respectively and O, the origin. We take each side of the smallest squares along with both axes as unit. Now, we plot the points $(-2,6),(0,3),(2,0)$, obtained from equation (1) on the graph paper and join them each other. The graph is a straight line. In the same way, we plot the points $(1,4),(2,0),(3,-4)$ obtained from equation (2) and join them each other. This graph is also a straight line. Let the two straight lines intersect at P. It is seen from the picture that the coordinates of the point of intersection are $(2,0)$.

$\therefore$ solution is $x=2$, or solution is 2

Activity : Find four points on the graph of the equation $2x-y-3=0$ in terms of a table. Then, taking unit of a fixed length on the graph paper, plot the points and join them each other. Is the graph a straight line ?

Exercise 12·3

Solve by graphs :

1. $3x+4y=14$
   $4x-3y=2$
2. $2x-y=1$
   $5x+y=13$
3. $2x+5y=1$
   $x+3y=2$
4. $3x-2y=2$
   $5x-3y=5$
5. $\frac{x}{2}+\frac{y}{3}=2$
   $2x+3y=13$
6. $3x+y=6$
   $5x+3y=12$

7. $3x+2y=4$
$3x-4y=1$

8. $\frac{x}{2}+\frac{y}{3}=3$
$x+\frac{y}{6}=3$

9. $3x+2=x-2$

10. $3x-7=3-2x$

12·5 Formation of simultaneous equations from real life problems and solution.

In everyday life, there occur some such mathematical problems which are easier to solve by forming equations. For this, from the condition or conditions of the problem, two mathematical symbols, mostly the variables x, y are assumed for two unknown expressions. Two equations are to be formed for determining the values of those unknown expressions. If the two equations thus formed are solved, values of he unknown quantities will be found.

Example 11. If 5 is added to the sum of the two digits of a number consisting of two digits, the sum will be three times the digits of the tens place. Moreover, if the places of the digits are interchanged, the number thus found will be 9 less than the original number. Find the number.

Solution : Let the digit of the tens place of the required number be x and its digits of the units place is $y.$ Therefore, the number is $10x+y.$

$\therefore$ by the 1st condition, $x+y+5=3x$.........(1)

and by the 2nd condition, $10y+x=(10x+y)-9$.......(2)

From equation (1), we get, $y=3x-x-5$, or $y=2x-5$........(3)

Again from equations (2), we get,

$10y-y+x-10x+9=0$

or $9y-9x+9=0$

or $y-x+1=0$

or $2x-5-x+1=0$ [putting the value of y from (3)]

or $x=4$

putting the value of x in (3), we get,

$y=2\times4-5$

$=8-5$

$=3$

$\therefore$ the number will be

$10x+y=10\times4+3$

$=40+3$

$=43$

$\therefore$ the number is 43

Example 12. 8 years ago, father's age was eight times the age of his son. After 10 years, father's age will be twice the age of the son. What are their present ages ?

Solution : Let the present age of father be x year and age of son is y year.

$\therefore$ by 1st condition $x-8=8(y-8)$........(1)

and by 2nd condition, $x+10=2(y+10)$.......(2)

From (1), we get, $x-8=8y-64$

or $x=8y-64+8$

or $x=8y-56$............(3)

From (2), we get, $x+10=2y+20$

or $8y-56+10=2y+20$ [putting the value of x from (3)]

or $8y-2y=20+56-10$

or $6y=66$

or $y=11$

$\therefore$ from (3), we get, $x=8\times11-56=88-56=32$

$\therefore$ at present, Father's age is 32 years and son's age is 11 years.

Example 13. Twice the breadth of a rectangular garden is 10 metres more than its length and perimeter of the garden is 100 metre.

a. Assuming the length of the garden to be x metre and its breadth to be y metre, form system of simultaneous equations.

b. Find the length and breadth of the garden.

c. There is a path of width 2 metres around the outside boundary of the garden. To make the path by bricks, it costs $110\cdot00$ per square metre. What will be the total cost ?

Solution : a. Length of the rectangular garden is x metre and its breadth is y metre.

$\therefore$ by 1st condition, $2y=x+10$........(1)

and by 2nd condition, $2(x+y)=100$.......(2)

y metre

x metre

b. From equation (1), we get, $2y=x+10$........(1)

From equation (2), we get, $2x+2y=100$.......(2)

or $2x+x+10=100$ [putting the value of $2y$ from (1)]

or $3x=90$

or $x=30$

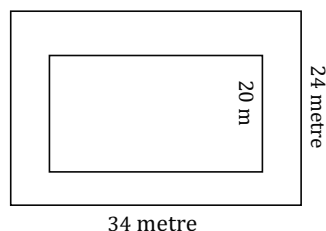
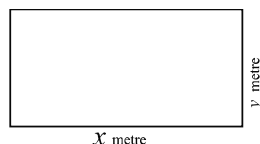
$\therefore$ from (1), we get, $2y=30+10$ [putting the value of x]

or, $2y=40$

or, $y=20$

$\therefore$ length of the garden is 30 metres and its breadth is 20 metres.

c. Length of the garden with the path is (30+4) metres.

= 34 metres

and breadth is (20+4) metres = 24 metres.

$\therefore$ Area of the path = Area of the graden with the path − area of the garden

$= (34 \times 24 - 30 \times 20)$ square metre

$= (816 - 600)$ square metre

$= 216$ square metre

$\therefore$ cost for making the path by bricks

= Tk. 216×110

= Tk. 23760

Activity : If in triangle ABC, $\angle B = 2x$ degree, $\angle C = x$ degree, $\angle A = y$ degree and $\angle A = \angle B + \angle C$, find the value of x and y.

Exercise 12·4

1. For which of the following conditions are the system of equations $ax + by + c = 0$ and $px + qy + r = 0$ consistent and mutually independent ?

   a. $\frac{a}{p} \neq \frac{b}{q}$ b. $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$ c. $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$ d. $\frac{a}{p} = \frac{b}{q}$

2. If $x + y = 4, x - y = 2$, which one of the following is the value of (x, y) ?

   a. $(2, 4)$ b. $(4, 2)$ c. $(3, 1)$ d. $(1, 3)$

3. If $x + y = 6$ and $2x = 4$, what is the value of y ?

   a. 2 b. 4 c. 6 d. 8

4. For which one of the following equation is the adjoining chart correct?

   a. $y = x - 4$ b. $y = 8 - x$
   c. $y = 4 - 2x$ d. $y = 2x - 4$

x	0	2	4
7	-4	0	4

5. If $2x - y = 8$ and $x - 2y = 4$, what is $x + y$?

   a. 0 b. 4 c. 8 d. 12

6. Observe the following information : :

   i. The equations $2x - y = 0$ and $x - 2y = 0$ are mutually dependent.
   ii. Graph of the equation $x - 2y + 3 = 0$ passes through the point $(-3, 0)$.
   iii. Graph of the equation $3x + 4y = 1$ is a straight line.

On the basis of information above, which one of the following is correct ?

   a. *i* and *ii* b. *ii* and *iii* c. *i* and *iii* d. *i*, *ii* and *iii*

7. Length of the floor of a rectangular r oom is 2 metres more than its breadth and perimeter of the floor is 20 metres.

Answer the following questions :

(1) What is the length of the floor of the room in metre ?

a. 10 b. 8 c. 6 d. 4

(2) What is the area of the floor of the room in square metre ?

a. 24 b. 32 c. 48 d. 80

(3) How much taka will be the total cost for decorating the floor with mosaic at Tk. 900 per square metre ?

a. 72000 b. 43200 c. 28800 d. 21600

Solve by forming simultaneous equations (8 - 15) :

8. If 1 is added to each of numerator and denominator of a fraction, the fraction will be $\frac{4}{5}$. Again, if 5 is subtracted from each of numerator and denominator, the fraction will be $\frac{1}{2}$. Find the fraction.
9. If 1 is subtracted from numerator and 2 is added to denaminator of a fraction, the fraction will be $\frac{1}{2}$. Again, if 7 is subtracted from numerator and 2 is subtracted from denominator, the fraction will be $\frac{1}{3}$. Find the fraction.
10. The digit of the units place of a number consisting of two digits is 1 more than three times the digit of tens place. But if the places of the digits are interchanged, the number thus found will be equal to eight times the sum of the digits. What is the number ?
11. Difference of the digits of a number consisting of two digits is 4. If the places of the digits are interchanged, sum of the numbers so found and the original number will be 110 ; find the number.
12. Present age of mother is four times the sum of the ages of her two daughters. After 5 years, mother’s age will be twice the sum of the ages of the two daughters. What is the present age of the mother ?
13. If the length of a rectangular region is decreased by 5 metres and breadth is increased by 3 metres, the area will be less by 9 square metres. Again, if the length is increased by 3 metres and breadth is increased by 2 metres, the area will be increasd by 67 square metres. Find the length and breadth of the rectangle.

14. A boat, rowing in favour of current, goes 15 km per hour and rowing against the current goes 5 km per hour. Find the speed of the boat and current.

15. A labourer of a garments serves on the basis of monthly salary. At the end of every year she gets a fixed increment. Her monthly salary beomes Tk. 4050 after 4 years and Tk. 6 after 8 years. Find the salary at the beginning of her service and amount of annual increment of salary.

16. A system of simple equations are

$$x + y = 10$$
$$3x - 2y = 0$$

a. Show that the equations are consistent. How many solutions do they have ?

b. Solving the system of equations, find (x, y).

c. Find the area of the triangle formed by the straight lines indicated by the equations with the x axis.

17. If 7 is added to the num. of a fraction, the value of the fraction is the integer 2. Again, if 2 is subtracted from the denominator, value of the fraction is the integer 1.

a. Form a system of equations by the fraction $\frac{x}{y}$.

b. Find (x, y) by solving the system of equations by the method of cross-multiplication. What is the fraction ?

c. Draw the graphs of the system of equations and verify the correctness of the obtained values of (x, y).